

Competitive solar heat engines

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Abstract

This article presents a technical innovation consisting of replacing the piston–cylinder system by metallic bellows in free-displacer Stirling engines. It deals with the advantage of heat engines of low power, integrating this innovation with plane solar collectors. A theoretical study requiring notions of finite time thermodynamics determines the optimised dimensioning of such systems and makes them appear serious competitors, on the one hand to solar cells to exploit solar energy and, on the other hand, to power stations regarding kW h unit cost.

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1. Introduction

Energy is the essential factor for higher standards of living. It is, therefore, absolutely vital in economically developed societies. Considered in the long term, the question of maintaining higher standards of living is intrinsically linked to hypothetical sources that can deliver energy in profusion, without a limit in time. It is easy to see that solar electromagnetic radiation provides such a source. Using an elementary calculation, we can see that the surface of a square of side 400 km, equipped to transform solar electromagnetic radiation into electrical energy with 0.15 efficiency, is sufficient to satisfy the energy requirements of the world, at the current level of approximately 10 TW. Thus considered, the question of long-term economic development meets the recommendations of the Harare declaration [1],

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Nomenclature

A	surface area [m^2]
c_p	specific heat at constant pressure [J/kg K]
c_v	specific heat at constant volume [J/kg K]
h	heat exchange coefficient [$\text{W/m}^2 \text{ K}$]
m	mass of driving medium [kg]
p	pressure [Pa]
Q	heat exchanged during a cycle [J]
$\dot{Q}$	heat power [W]
r	mass constant of gases [J/kg K]
S	entropy [J/K]
T	temperature [K]
U	internal energy [J]
V	volume [m^3]
W	work exchanged during a cycle [J]
$\dot{W}$	mechanical power [W]

Greek symbols

α	regenerator effectiveness
ϕ	flux heat [W/m^2]
η	efficiency

Indexes

abs	absorber
amb	ambient atmosphere
h	hot source for driving medium
coll	collector
hot	hot source for engine
c	cold source for driving medium
cold	cold source for engine
lk	heat leakages
fric	viscous or static friction
g	global
M	absorber boundary value, see Eq. (9)
max	driving medium in isothermal expansion
min	driving medium in isothermal compression
mot	motor
opt	optimal
reg	regenerator
S	solar
1, 2, 3, 4	points of the cycle

Intermediary variables ρ, ρ_0 Eq. (12) a_0 Eq. (19) b_0, b_1, b_2 Eq. (21) K, c_0, c_1, c_2 Eq. (22) α_1, β_1 Eq. (25) α_2, β_2 Eq. (26)

advocating the research, development and implementation of technical systems for the massive exploitation of solar energy.

The value mentioned above of 0.15 corresponds to the average efficiency of solar cells, systems generally chosen to exploit solar electromagnetic radiation. The efficiency of a cell is, more accurately, 0.12 regarding commercial quality, 0.17 for improved quality and 0.24 for the best cells manufactured in laboratories. These values, all notably inferior to those of heat engine efficiency used in conventional energy production, would nevertheless be satisfactory within the context of exploiting solar energy—exploiting any other form of energy originating from solar energy (wind, hydraulic or biomass) does indeed give derisory final efficiency, inferior to 0.001—but ultimately, it is essentially for economically competitive reasons with regard to conventional energy production that solar cells are not more widely used. Following this line of thinking, the heat option for exploiting solar electromagnetic radiation should be taken into consideration as the technology of heat engines allows for moderate cost prices and their conception, today, could benefit from solid theoretical knowledge (consult the works by Walker [2], Organ [3], Senft [4] and Bejan [5]), when combined with sustained creativity (see, for example, the patents of Lamos [6], Bliesner [7] and Kinnersly [8]).

The author of this article would like to make credible the idea of exploiting the field of solar electromagnetic radiation using the heat option, by publishing a technical innovation, apparently entirely original in bibliographies and patent registrations. This innovation consists of replacing the traditional piston-cylinder system in the Stirling cycle heat engine with metallic bellows; the main obstacle to achieving very high efficiency with heat engines from 100 to 3000 W of power therefore being eliminated, it becomes necessary to study what level of optimal performance the combination, composed of such a heat engine and a solar collector, could reasonably attain.

The working principle of the system is justified and described in Section 2. The mathematical model is set out in Section 3 and optimal dimensioning is determined in Section 4. The numerical application in Section 5 enables the most sensitive technical points to be defined and tackles, finally, the question of economic competitiveness in terms of the cost of kW h produced.

2. Solar heat engines studied

2.1. Preliminary on Carnot efficiency cycles

The Carnot cycle is composed of isothermal compression (1–2) at temperature $T_{\min}$, adiabatic compression (2–3), isothermal expansion (3–4) at temperature $T_{\max}$ and adiabatic expansion (4–1) (Fig. 1). When it occurs under infinitesimal temperature differences, the temperatures $T_{\max}$ and $T_{\min}$ become identified with the temperatures T_h and T_c of the heat sources, and in the absence of friction, the efficiency of the cycle attains the well-known upper boundary value η_{Carnot} , dependent only on the temperatures of the hot and cold sources:

$$\eta_{\text{Carnot}} = 1 - \frac{T_c}{T_h}. \quad (1)$$

By explaining the terms involved in defining the efficiency of an engine $\eta_{\text{mot}} = -W/Q_{\text{hot}}$, (1) can be established. Using first principles, the reverse of the indicated mechanical work, exchanged during a cycle, can be written:

$$-W = Q_{\text{hot}} + Q_{\text{cold}} \quad (2)$$

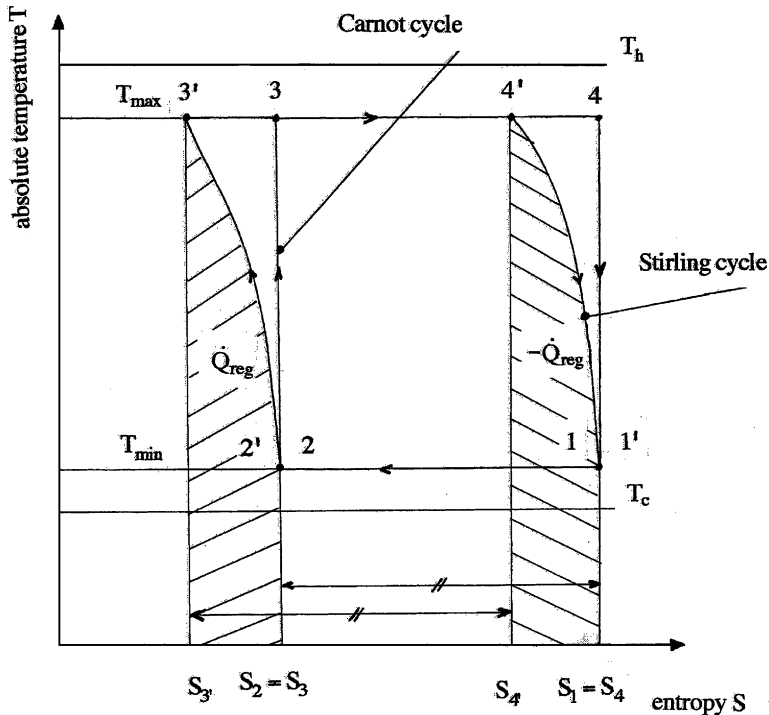


Fig. 1. The Carnot cycle and Stirling cycle.

and we can express the efficiency, written according to the heat exchanges with the heat sources, as:

$$\eta_{\text{mot}} = \frac{Q_{\text{hot}} + Q_{\text{cold}}}{Q_{\text{hot}}}. \quad (3)$$

Relation (3) then leads Eq. (1), using second principles: the heat values Q_{hot} and Q_{cold} are replaced, exchanged by the driving medium during a reversible cycle, by $T_h \cdot (S_4 - S_3)$ and $T_c \cdot (S_2 - S_1)$, respectively, and it can be noted that $-(S_2 - S_1) = S_4 - S_3$ in the Carnot cycle.

The existence of an infinite number of cycles reaching Carnot cycle efficiency, or Stirling cycles, is well-known (see, for example, [9]). It can be observed by remarking that all relations of the form

$$\eta_{\text{mot}} = \frac{|Q_{\text{hot}} - Q_{\text{reg}} + Q_{\text{cold}} + Q_{\text{reg}}|}{Q_{\text{hot}}} \quad (4)$$

replacing Eq. (3) lead to Eq. (1) using the calculation indicated above. The terms $-Q_{\text{reg}}$ and Q_{reg} may be considered as the heat exchanged during the evolutions (4–1) and (2–3), and verifying by definition, for example:

$$Q_{\text{reg}} = \int_2^3 T \, dS = - \int_4^1 T \, dS. \quad (5)$$

As the Q_{reg} and $-Q_{\text{reg}}$ exchanges take place without disturbing the heat exchanges with the hot and cold sources, the heat value Q_{reg} must be stored and destocked in an intermittent manner within an annexed device, the regenerator.

Finally, Stirling cycles are composed of a compression (1–2) and expansion (3–4), both isothermals and verifying $-(S_2 - S_1) = S_4 - S_3$, as well as the two intermediary phases (2–3) and (1–4), whose evolution is only limited in that the heat $|Q_{\text{reg}}|$, exchanged with the regenerator, verifies (5) (Fig. 1).

2.2. Description of the innovative solar heat engine

The objective of this article is to determine the optimal performances that a competitive solar heat engine can realistically reach, on the energy and economic planes.

Solar electromagnetic radiation is collected using a flat plate solar collector, which is easier to produce than a concentrating collector. The heat power, originating from the solar electromagnetic radiation and transmitted to the driving medium through an absorber, is transformed into mechanical power by a heat engine, whose theoretical cycle attains Carnot efficiency.

The succession of isentrops and isotherms in the Carnot cycle requires the conditions of heat exchange to be modified several times per cycle. Technically, this leads to complicated systems and, to ensure the reliability and economic competitiveness of the solar heat engine, in this case, a free-displacer Stirling engine was chosen. The configuration of a free piston and regenerator was adopted, which, according Rochelle [10], is the most appropriate.

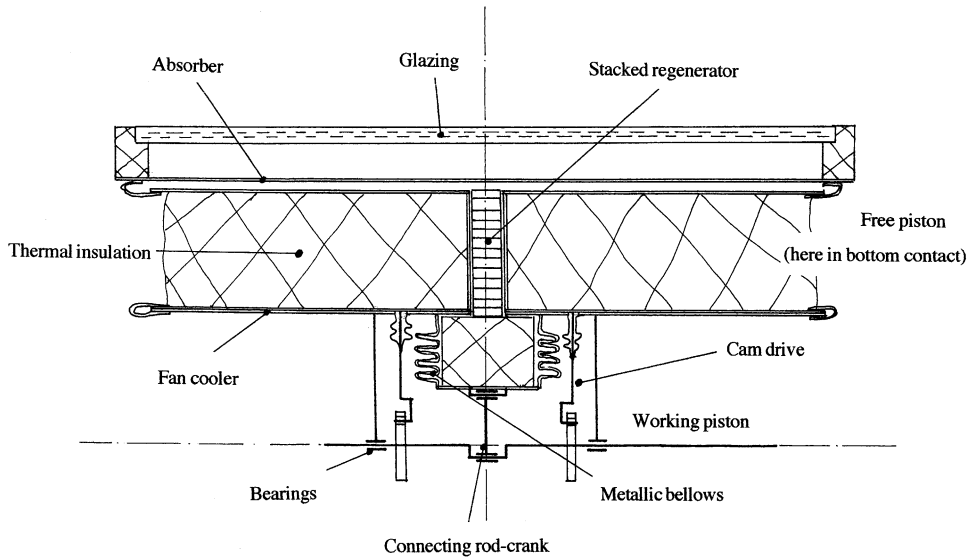


Fig. 2. Diagram of the innovative solar heat engine.

The practical arrangements which bring the true efficiency of the heat engine close to theoretical efficiency are shown in Fig. 2. The three main points are as follows:

1. The piston-cylinder system is replaced by metallic bellows, reducing the mechanical friction to a negligible value,
2. The regenerator, interdependent with the working piston, has a weak flow area with regard to heat exchange surface; it is formed from a stack of air-permeable plates, with heat insulation between them, so the storage and destocking of heat takes place under the temperature gradient imposed by the absorber and fan cooler,
3. The bases of the cylinder, absorber and fan cooler have a large surface area, so heat exchange can take place with only slight differences in temperature.

3. Mathematical model

3.1. Hypotheses

The mathematical model is a synthesis of those by Magot-Cuvru [9] and Badescu et al. [11]. Similar to the latter, the evolutions (1–2) and (3–4), taken to be reversible isothermals, take into account the difference in temperature between the sources and the driving medium. The absorber acts as the hot source; thus, $T_h = T_{abs}$. The cold source for the engine is the ambient atmosphere; $T_c = T_{amb}$ is obtained. Finally, in normal working conditions, we obtain $T_{min} > T_{amb}$ and $T_{max} < T_{abs}$.

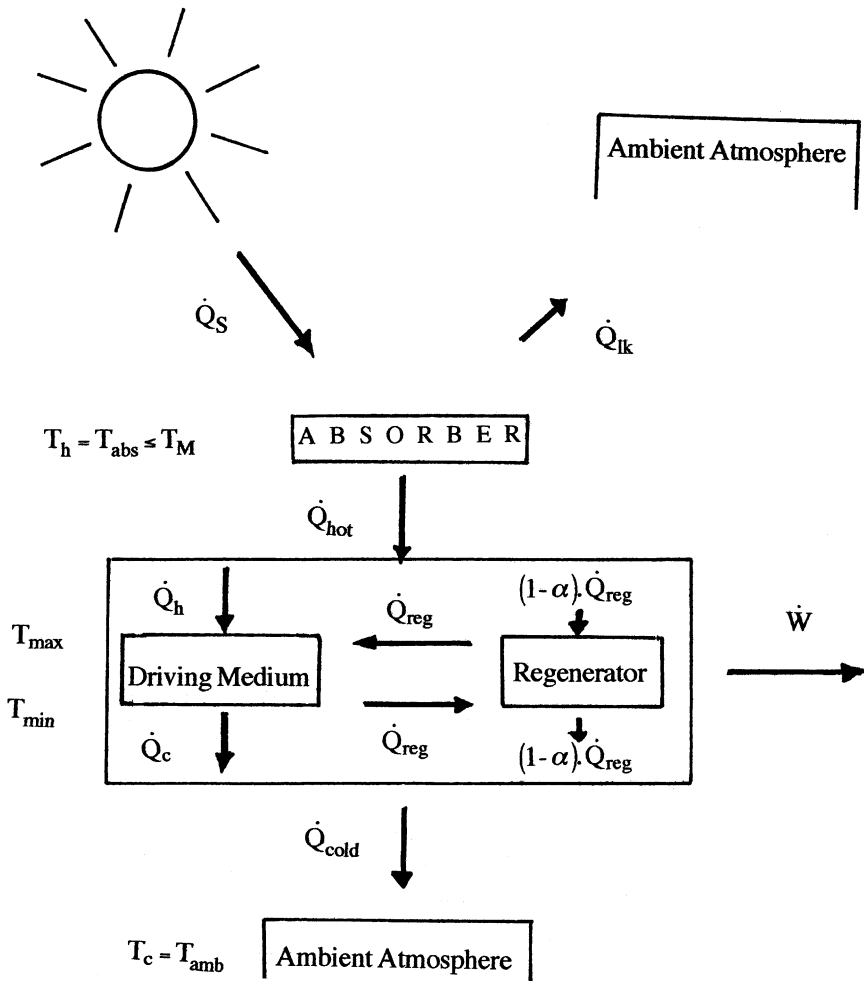


Fig. 3. Modelling of energy exchanges.

(Fig. 1). The driving medium behaves as a perfect gas of sensitive specific heat values c_v and c_p and mass constant of gas r .

The energy exchanges take place at a stationary rate of flow. The fluxes taken into account are indicated in diagrammatic form in Fig. 3. All the fluxes exchanged outside the engine are modelled in the same way as by Badescu et al. [11]. The heat power $\dot{Q}_S$, originating from the solar electromagnetic radiation, is redistributed by the absorber, for $\dot{Q}_{hot}$, to the engine, and for $\dot{Q}_{lk}$, into the ambient atmosphere, in the form of convective heat leakages; we obtain

$$\dot{Q}_S + \dot{Q}_{hot} + \dot{Q}_{lk} = 0. \quad (6)$$

The heat power $\dot{Q}_{lk}$ includes the leakages through the side of the collector exposed to the sun and by the side in contact with the inside of the engine. The heat power discharged by the engine into the outside medium is $\dot{Q}_{cold}$.

The fluxes of heat internal to the engine are represented with the help of the Stirling cycle imperfect exchange model, by Magot-Cuvru [9]. The heat power retained, $\dot{Q}_{hot}$, is divided into $\dot{Q}_h$, supplied to the driving medium, and $(1 - \alpha)|\dot{Q}_{reg}|$, supplied to the regenerator. The power discharged, $\dot{Q}_{cold}$, includes the power $\dot{Q}_c$ originating from the driving medium and $(1 - \alpha)|\dot{Q}_{reg}|$ from the regenerator. The regenerator efficiency α is such that $(1 - \alpha)|\dot{Q}_{reg}|$ represents the heat originating from the hot source, passing through the regenerator and discharged into the cold source, for each cycle. The driving medium, which retains $\dot{Q}_h$ and supplies $\dot{Q}_c$, also supplies the mechanical power indicated by $\dot{W}$ and the dissipated one $\dot{W}_{fric}$. Simultaneously, during a cycle, it retains from and gives to the regenerator, the power $\dot{Q}_{reg}$.

The particular arrangements relative to the Stirling cycle must now be specified. During the intermediary evolutions (2–3) and (4–1), the pressure evolution is taken as being affine according to the volume, as it is linear for the Bouasse cycle. This hypothesis amounts to interpolating linearly, in the (p, V) diagram, the evolution between points 2 and 3, on the one hand, and points 4 and 1 on the other. It is simple to verify that this modelling respects condition (5) on regeneration, when the condition of the Stirling cycles $-(S_2 - S_1) = S_4 - S_3$ has been imposed. This modelling was the one chosen to facilitate further calculations.

Finally, dissipation is taken into account. The mechanical work dissipated through the forces of viscosity acting in the driving medium flow and the forces of solid friction acting on the mechanisms is taken into account by the efficiency η_{mec} , defined by

$$\eta_{mec} = \frac{|\oint p \, dV| - |W_{frot}|}{|\oint p \, dV|},$$

where $|\oint p \, dV|$ represents the mechanical work of the reversible cycle.

3.2. Global efficiency of the solar heat engine

The global efficiency of the solar heat engine η_g can be written as

$$\eta_g = \eta_{coll} \cdot \eta_{mot}. \quad (7)$$

The definition of solar collector efficiency (for example, [12: p. 411]) and Eq. (6) enable us to write $\eta_{coll} = 1 - |\dot{Q}_{lk, coll}|/\dot{Q}_S$. By explaining the heat power values with regard to the fluxes of heat $\phi_S = h_{lk} \cdot (T_M - T_{amb})$ and $\phi_{lk} = h_{lk} \cdot (T_{abs} - T_{amb})$, it can be deduced that

$$\eta_{coll} = 1 - \frac{T_{abs} - T_{amb}}{T_M - T_{amb}}, \quad (8)$$

with

$$T_M = T_{\text{amb}} + \frac{\phi_S}{h_{lk}} \quad (9)$$

for the absorber boundary temperature, obtained when $\phi_S = \phi_{lk}$.

To express the engine efficiency, it must be noted that $\dot{Q}_{\text{hot}}$ is expressed, in the present case of the Stirling cycle, as $\dot{Q}_{\text{hot}} = \dot{Q}_h + (1 - \alpha)|\dot{Q}_{\text{reg}}|$, and $\dot{Q}_{\text{cold}}$ is given by $\dot{Q}_{\text{cold}} = \dot{Q}_c - (1 - \alpha)|\dot{Q}_{\text{reg}}|$ (see Fig. 3). Thus, Eq. (3) expressing η_{mot} is given as

$$\eta_{\text{mot}} = \frac{\dot{Q}_h + \dot{Q}_c}{\dot{Q}_h + (1 - \alpha)|\dot{Q}_{\text{reg}}|}.$$

With the hypothesis of isotherm evolution reversibility, $\dot{Q}_h = T_{\text{max}} \cdot (S_4 - S_3)$ and $\dot{Q}_c = T_{\text{min}} \cdot (S_2 - S_1)$ can be noted. First principles applied to the evolution (2–3) enables one to write $\dot{Q}_{\text{reg}} = U_3 - U_2 + \int_2^3 p \, dV$, which, with the hypothesis of the law of sure evolution, becomes

$$\dot{Q}_{\text{reg}} = mc_V(T_{\text{max}} - T_{\text{min}}) + \frac{p_3 + p_2}{2} \cdot (V_3 - V_2). \quad (10)$$

In addition, the variations of entropy, in the case of the present driving medium, can be expressed as $S_4 - S_3 = m \ln(V_4/V_3)$ and $S_1 - S_2 = m \ln(V_1/V_2)$ and this leads to

$$\eta_{\text{mot}} = \frac{T_{\text{max}} \ln \frac{V_4}{V_3} - T_{\text{min}} \ln \frac{V_1}{V_2}}{T_{\text{max}} \left[\ln \frac{V_4}{V_3} + \frac{1 - \alpha}{\gamma - 1} + \frac{(1 - \alpha)}{2} \left(1 - \frac{V_2}{V_3} \right) \right] - T_{\text{min}} \left[\frac{1 - \alpha}{\gamma - 1} - \frac{(1 - \alpha)}{2} \left(\frac{V_3}{V_2} - 1 \right) \right]}. \quad (11)$$

Finally, taking the condition of Stirling cycles $S_4 - S_3 = S_1 - S_2$ into account, by writing $\rho = V_4/V_3 = V_1/V_2$ and $\rho_0 = V_1/V_3$, we obtain

$$\eta_{\text{mot}} = \frac{T_{\text{max}} \ln \rho - T_{\text{min}} \ln \rho}{T_{\text{max}} \left[\ln \rho + \frac{1 - \alpha}{\gamma - 1} + \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho} \right] - T_{\text{min}} \left[\frac{1 - \alpha}{\gamma - 1} - \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho_0} \right]}. \quad (12)$$

The final stage consists in taking the irreversibility values into account. As they only appear due to dissipation and they only affect the work and not the exchanged heat values, it is sufficient to multiply by the mechanical efficiency $\eta_{\text{méc}}$. The efficiency can be finally expressed as

$$\eta_{\text{mot}} = \eta_{\text{méc}} \frac{T_{\text{max}} \ln \rho - T_{\text{min}} \ln \rho}{T_{\text{max}} \left[\ln \rho + \frac{1 - \alpha}{\gamma - 1} + \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho} \right] - T_{\text{min}} \left[\frac{1 - \alpha}{\gamma - 1} - \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho_0} \right]}. \quad (13)$$

By substituting (8) and (13) in (7), we can express the global efficiency η_g of the solar heat engine:

$$\eta_g = \eta_{\text{méc}} \frac{\left(1 - \frac{T_{\text{abs}} - T_{\text{amb}}}{T_{\text{M}} - T_{\text{amb}}}\right) (T_{\text{max}} \ln \rho - T_{\text{min}} \ln \rho)}{T_{\text{max}} \left[\ln \rho + \frac{1 - \alpha}{\gamma - 1} + \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho} \right] - T_{\text{min}} \left[\frac{1 - \alpha}{\gamma - 1} - \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho_0} \right]} \quad (14)$$

3.3. Heat power exchanged with the sources

The value of engine power is entirely determined by the heat power values of $\dot{Q}_{\text{hot}}$ and $\dot{Q}_{\text{cold}}$. The latter are imposed by the laws of heat exchange in the driving medium with the hot and cold sources. It is accepted that the heat exchanges are indeed represented by the laws of Newton, with coefficient heat transfer h_{hot} and h_{cold} , respectively, taken to be constant.

The heat power supplied by the absorber is thus $\dot{Q}_{\text{hot}} = A_{\text{abs}} \cdot h_{\text{hot}}(T_{\text{abs}} - T_{\text{max}})$. By substituting this expression of $\dot{Q}_{\text{hot}}$ in (6) and by explaining $\dot{Q}_{\text{S}}$ in an analogical way, we obtain

$$(T_{\text{M}} - T_{\text{abs}}) = \frac{h_{\text{hot}}}{h_{\text{lk}}} (T_{\text{abs}} - T_{\text{max}}). \quad (15)$$

Likewise, the heat power lost by the engine is $\dot{Q}_{\text{cold}} = A_{\text{cold}} \cdot h_{\text{cold}}(T_{\text{amb}} - T_{\text{min}})$. In addition, relation (3) leads to $\dot{Q}_{\text{cold}} = -(1 - \eta_{\text{mot}})\dot{Q}_{\text{hot}}$. By using (15) to eliminate T_{max} , we obtain

$$(1 - \eta_{\text{mot}}) \cdot (T_{\text{M}} - T_{\text{abs}}) = \frac{h_{\text{cold}} \cdot A_{\text{cold}}}{h_{\text{lk}} \cdot A_{\text{abs}}} (T_{\text{min}} - T_{\text{amb}}). \quad (16)$$

4. Optimised solar heat engines

4.1. Principle of optimisation

Optimising the dimensions of the solar heat engine requires notions of finite time thermodynamics, described by Feidt [12].

The problem being treated contains variables, values for which dimensioning occurs, in this case T_{abs} , T_{max} , T_{min} , ρ , ρ_0 and α . The variables that are independent of dimensioning, h_{hot} , h_{cold} , h_{lk} , ϕ_{S} , T_{amb} and, according to (9), T_{M} are considered as parameters.

The problem consists in expressing, according to the parameters, the variables that maximise global efficiency, given by (14), under the constraints in (15) and (16). Studying the partial derivatives of η_g , established on the basis of (14), easily demonstrates that the efficiency is a monotonous function of the variables ρ , ρ_0 and α . Optimising according to these values consists, therefore, in using the smallest value or the largest concretely permitted value, according to whether η decreases or increases with the considered variable.

The mathematical form of the constraint in (16) makes application of Lagrange's method of multipliers difficult. For this reason, optimising η_g according to T_{abs} was studied directly which first required T_{max} and T_{min} to be eliminated from (14) using (15) and (16). The optimised dimensioning of the solar engine corresponds to the variables relative to the value of T_{abs} which makes η_g maximum.

4.2. Expression of efficiency in conditions close to optimum

For easier calculation, it is useful to introduce the variable

$$X = \frac{T_{\text{abs}}}{T_{\text{M}}}, \quad (17)$$

and reason using the reduced temperatures of $T_{\text{max}}/T_{\text{M}}$ and $T_{\text{min}}/T_{\text{M}}$. With these notations, (15), which can be written as

$$\frac{T_{\text{max}}}{T_{\text{M}}} = (1 + a_0)X - a_0, \quad (18)$$

by writing

$$a_0 = \frac{h_{\text{lk}}}{h_{\text{ch}}}, \quad (19)$$

supplies the relation giving $T_{\text{max}}/T_{\text{M}}$ according to X .

Relation (16) brings the factor $1 - \eta_{\text{mot}}$ into play. By replacing η_{mot} with the expression at (13), η_g appears as a rational fraction in X having a degree-4 polynomial as the numerator as well as the denominator, which notably complicates the expression of the maximum. To get around this difficulty, we can suppose that the final result will not be significantly removed from the result obtained in [12: p 333]; we can, therefore, admit at this stage in the calculation that $1 - \eta_{\text{mot}} \approx \sqrt{T_{\text{amb}}/T_{\text{M}}}$. With this simplification, (16) can be written as

$$\frac{T_{\text{min}}}{T_{\text{M}}} = -b_0b_1X + (b_0b_1 + b_2) \quad (20)$$

by writing

$$\begin{cases} b_0 = \frac{h_{\text{lk}} \cdot A_{\text{abs}}}{h_{\text{cold}} \cdot A_{\text{cold}}} \\ b_1 = 1 - \eta_{\text{mot}} \approx \sqrt{\frac{T_{\text{amb}}}{T_{\text{M}}}} \\ b_2 = \frac{T_{\text{amb}}}{T_{\text{M}}} \end{cases} \quad (21)$$

Using the notations in (21) and by writing

$$\begin{cases} K = \frac{\eta_{\text{méc}}}{1 - b_2} \\ c_0 = \ln \rho \\ c_1 = \frac{1 - \alpha}{\gamma - 1} + \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho} \\ c_2 = \frac{1 - \alpha}{\gamma - 1} - \frac{(1 - \alpha)(\rho - \rho_0)}{2\rho_0} \end{cases} \quad (22)$$

the expression of global efficiency η_g in (14) can also be written as

$$\eta_g = Kc_0(1 - X) \frac{\frac{T_{\text{max}}}{T_M} - \frac{T_{\text{min}}}{T_M}}{(c_0 + c_1) \frac{T_{\text{max}}}{T_M} - c_2 \frac{T_{\text{min}}}{T_M}}, \quad (23)$$

and the elimination of the intermediary variables T_{max}/T_M and T_{min}/T_M using the relations in (18) and (20) finally supplies

$$\eta_g = K(1 - X) \frac{(\alpha_1 X - \beta_1)}{(\alpha_2 X - \beta_2)}, \quad (24)$$

where

$$\begin{cases} \alpha_1 = 1 + a_0 + b_0 b_1 \\ \beta_1 = a_0 + b_0 b_1 + b_2 \end{cases} \quad (25)$$

and

$$\begin{cases} \alpha_2 = \left(1 + \frac{c_1}{c_0}\right)(1 + a_0) + \frac{c_2}{c_0} b_0 b_1 \\ \beta_2 = \left(1 + \frac{c_1}{c_0}\right)a_0 + \frac{c_2}{c_0} b_0 b_1 + \frac{c_2}{c_0} b_2 \end{cases} \quad (26)$$

were written. Using these notations, the engine efficiency η_{mot} can be written as

$$\eta_{\text{mot}} = \eta_{\text{méc}} \frac{(\alpha_1 X - \beta_1)}{(\alpha_2 X - \beta_2)}. \quad (27)$$

The optimal temperature T_{abs} can be found using Eq. (24). The extremum condition $d\eta_g/dX = 0$ leads to the maximum in X , given by the relation

$$X^2 - 2 \frac{\beta_2}{\alpha_2} X + \left(\frac{\beta_2}{\alpha_2} - \frac{\beta_1}{\alpha_1} + \frac{\beta_1}{\alpha_1} \frac{\beta_2}{\alpha_2} \right) = 0. \quad (28)$$

As $\beta_1/\alpha_1 > \beta_2/\alpha_2$ and $1 > \beta_2/\alpha_2$, the root to be retained can be expressed as

$$X_{\text{opt}} = \left(\frac{T_{\text{abs}}}{T_M} \right)_{\text{opt}} = \sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1}{\alpha_1} - \frac{\beta_2}{\alpha_2}} + \frac{\beta_2}{\alpha_2}, \quad (29)$$

and gives, transferred in (24), the optimal efficiency relative to this approximate calculation:

$$\eta_{g,opt} = K \frac{\alpha_1}{\alpha_2} \times \left(1 - \sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2} + \frac{\beta_2}{\alpha_2}} \right) \frac{\sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2} + \frac{\beta_2}{\alpha_2}}}{\sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2}}} \quad (30)$$

By substituting in Eqs. (29), (18) and (20), we obtain

$$\begin{cases} T_{abs} = T_M \left(\sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2} + \frac{\beta_2}{\alpha_2}} \right) \\ T_{max} = T_M \left[(1 + a_0) \left(\sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2} + \frac{\beta_2}{\alpha_2}} \right) - a_0 \right] \\ T_{min} = T_M \left[-b_0 b_1 \left(\sqrt{1 - \frac{\beta_2}{\alpha_2}} \sqrt{\frac{\beta_1 - \beta_2}{\alpha_1} - \frac{\beta_2}{\alpha_2} + \frac{\beta_2}{\alpha_2}} \right) + (b_0 b_1 + b_2) \right] \end{cases} \quad (31)$$

4.3. Calculation of performances

Eqs. (13)–(16) of the model take into account the most significant imperfections: imperfect regenerator and heat transfers carried out with non-nil temperatures differences. Optimisation leading to the formulas in (30) and (31), based on the approximation $1 - \eta_{mot} \approx \sqrt{T_{amb}/T_M}$, rests on the hypothesis that the engine reaches Carnot efficiency. Numerically, we can compensate for this hypothesis using an iterative calculation, based on a fixed point method.

This iterative calculation assumes b_1 as an entry variable and $1 - \eta_{mot}$ as an exit variable. It is carried out for the value $X = X_{opt}$ given by (29). It consists in calculating $1 - \eta_{mot}$, according to b_1 , using Eq. (27), and the variables α_1 , α_2 , β_1 and β_2 depending on b_1 , having been previously calculated with the help of (25) and (26). The initial evaluation of b_1 is carried out, in the context of the approximate optimisation, using Eq. (21). The numerical applications carried out show that the method converges very rapidly.

The value $X = X_{opt}$ given by (29) contains the hypothesis $1 - \eta_{mot} \approx \sqrt{T_{amb}/T_M}$ and consequently, does not correspond to the maximum value for the global efficiency η_g . It is possible, without any risk of divergence, to apply the above iterative method for arbitrary values of X quite close to X_{opt} and thus search, by successive approximations, for the value of X which makes the global efficiency η_g maximum. Such a calculation gives the exact optimal characteristics.

5. Example of a competitive solar heat engine

5.1. Data

The collecting area is fixed at $S_{\text{abs}} = 1 \text{ m}^2$. The solar heat engine functions with a solar flux of heat $\phi_S = 800 \text{ W/m}^2$, in ambient atmosphere at the temperature $T_{\text{amb}} = 293 \text{ K}$. The choice of air as the driving medium leads us to adopt $r = 287 \text{ J/kg K}$ and $\gamma = 1.4$.

By manufacturing the regenerator according to the model indicated in Section 2.2, effectiveness α close to 1 may be obtained by increasing the number of stages. Calculations, which will soon be published, demonstrate that the value of $\alpha = 0.9$ is a realistic one. The total rate of compression $\rho_0 = V_1/V_3$ cannot reach very high values in free-displacer engines. The value of $\rho_0 = 3$ is compatible with the above value of α . The rate of compression ρ , linked to the duration of the intermediary phases, is fixed at 2.8.

In alternative conventional engines, a significant proportion of the friction originates from solid friction between the piston segments and the cylinder. In these engines, friction is unavoidable as it ensures air-tightness. It should be noted that power lost through friction increases with reaming and the power produced with capacity, so the piston-cylinder solution is only of value with sufficiently large engines of unitary power. The innovative solution in Section 2.2 completely eliminates this inconvenience, in so much as friction is reduced to that created in the mechanical system of movement transformation and in the driving medium internal flows. As all this friction diminishes with the rotational speed of the engine, it is possible to fix it at a low value, giving very good mechanical efficiency. The value of $\eta_{\text{méc}} = 0.9$, with the preceding values and global efficiency of approximately 0.15, leads to dissipated power of the order of 12 W, which is realistic, even if slightly overestimated.

The driving medium thicknesses on the absorber and cooler are of the order of a few millimeters, as a result of the surfaces of largest heat exchange in view of the volumes, of the order of liters, generated by the free piston. The thermal resistance of the absorber being essentially due to the thickness of the air, we are led to adopt $h_{\text{hot}} = 50 \text{ W/m}^2 \text{ K}$ using a simple calculation of transference by conduction. Regarding the cooler, the resistance due to the natural convection on the fan is added to this thermal resistance. The value of $h_{\text{cold}} = 10 \text{ W/m}^2 \text{ K}$ is adopted. The load piston, located on the fan side, reduces the exchange area A_{cold} . We accept $A_{\text{cold}} = 0.85 \text{ m}^2$.

5.2. Results and comments

The methods of calculation in Section 4.3, giving the optimal values of the absorber temperature T_{abs} , global efficiency η_g and the temperatures T_{max} and T_{min} , are applied to the numerical data above. We chose to study the variation of these variables according to h_{lk} , as it is the value that can be directly interpreted in technical terms, and therefore, economic competitiveness.

Table 1
Evolution of values with h_{lk} . Approximate calculation

h_{lk} [W/m ² °C]	5.46	2.73	1.37	0.91	0.68	0.55	0.46
$\phi_S/h_{lk}T_{amb}$	0.5	1	2	3	4	5	6
T_M [°C]	166.5	313.0	603.9	899.1	1196.5	1474.5	1759.1
T_{abs} [°C]	117.9	180.4	273.7	349.4	415.1	470.1	521.6
T_{max} [°C]	112.6	173.1	264.7	339.5	404.5	459.0	510.3
T_{min} [°C]	45.5	50.1	50.8	49.4	47.9	46.6	45.4
η_g	0.050	0.106	0.186	0.240	0.280	0.309	0.334
η_{mot}	0.151	0.234	0.328	0.384	0.422	0.448	0.469
η_{Carnot}	0.250	0.354	0.465	0.529	0.574	0.606	0.631

Table 1 gives the numerical results obtained by applying Eqs. (30) and (31), relative to approximate optimisation. Table 2 gives the results obtained using the iterative method applied with a calculated absorber temperature of T_{abs} , according to the approximate optimisation, using Eq. (29). Table 3 demonstrates several exact calculations obtained using the method of optimisation through successive approximations. Finally, Fig. 4 represents the evolution of working efficiency and the loss coefficient of h_{lk} , and Fig. 5, that of temperatures.

Figs. 4 and 5 comply with the fact that the absorber temperature and efficiency increase as the heat leakages diminish. For the values of $h_{lk} > 0.46$ W/m² K, the obtained values of T_{abs} correspond to the electromagnetic radiation of the absorber in the visible domain. As the real heat leakages are, therefore, no longer compatible with those anticipated, taken to be without loss by radiation, the evolution is only represented up to $h_{lk} > 0.46$ W/m² K.

The numerical results reveal that the efficiency of the Stirling engine studied is always between 60% and 74% of the Carnot efficiency, whereas the global efficiency is, due to the heat leakages from the solar collector, between 20% and 53% of the Carnot efficiency.

The values of h_{lk} , which enable solar cell efficiencies of 0.12, 0.17 and 0.24 to be reached, are 2.5, 1.55 and 0.9 W/m² K, respectively. The efficiencies of 0.12 and 0.17 for ordinary solar cells can, therefore, be attained by solar engines of the type studied, when they are equipped with relatively usual solar collectors, whereas the efficiency of 0.24 requires flat plate collectors giving an improved, but not

Table 2
Evolution of values with h_{lk} . Iterative calculation to the approximate optimum

h_{lk} [W/m ² °C]	5.46	2.73	1.37	0.91	0.68	0.55	0.46
T_M [°C]	166.5	313.0	603.9	899.1	1196.5	1474.5	1759.1
T_{abs} [°C]	117.9	180.4	273.7	349.4	415.1	470.1	521.6
T_{max} [°C]	112.6	173.1	264.7	339.5	404.5	459.0	510.3
T_{min} [°C]	46.6	52.8	56.2	56.8	56.7	56.5	56.2
η_g	0.049	0.104	0.181	0.235	0.274	0.303	0.327
η_{mot}	0.149	0.229	0.321	0.375	0.413	0.439	0.459
η_{Carnot}	0.250	0.354	0.465	0.529	0.574	0.606	0.631

Table 3
Evolution of values with h_{lk} . Precise calculation

h_{lk} [W/m ² °C]	5.46	0.91	0.46
T_M [°C]	166.5	899.1	1759.1
T_{abs} [°C]	120.2	367.6	553.1
T_{max} [°C]	115.1	357.9	541.9
T_{min} [°C]	45.1	54.8	54.5
η_g	0.049	0.235	0.327
η_{mot}	0.156	0.389	0.472
η_{Carnot}	0.255	0.543	0.645

exceptional, performance. Finally, if we consider, as the numerical example given by Pasquetti [13: p. 7] leads us to believe, that the value $h_{lk} = 0.50$ W/m² K is technically acceptable, then the graph in Fig. 5 indicates that a global efficiency of 0.31 may be reached. This greatly exceeds the maximal efficiency of 0.24 for solar cells.

To finish, we can verify that the approximate calculation supplies efficiency values less than 4% of the exact value and the iterative calculation, less than 0.01%. Approximate optimisation leads us to underestimate the temperature: from 3 to 30 °C for the absorber temperatures and maximal for the driving medium, from 1 and 10 °C for the driving medium minimum temperatures.

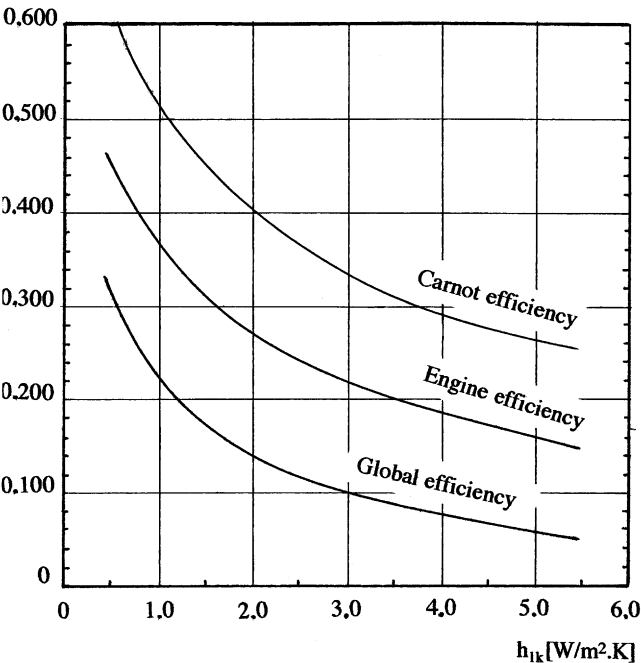


Fig. 4. Evolution of efficiency with heat leakages. Example of Section 5.1.

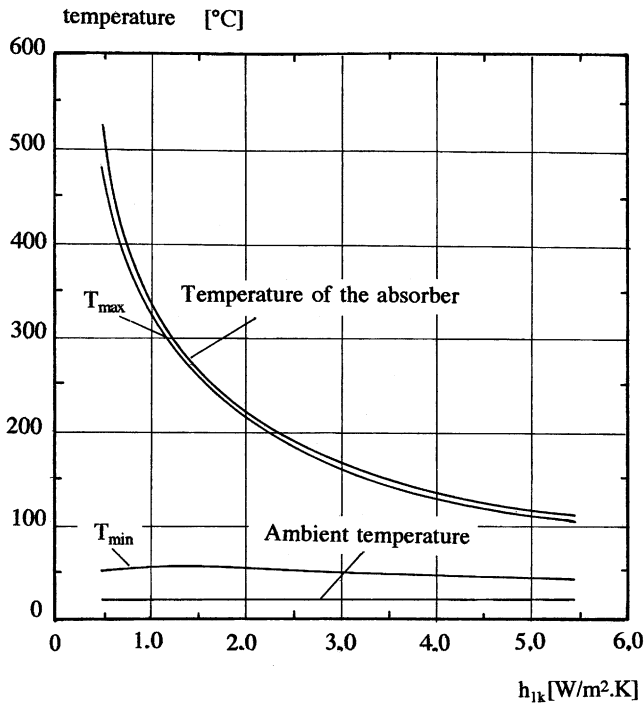


Fig. 5. Evolution of temperatures with heat leakages. Example of Section 5.1.

5.3. Evaluation of $kW h$ costing

The production of a first prototype with a collector area of $1 m^2$ at the GREEN has enabled the cost of a solar heat engine according to the model in Section 2.2 to be estimated. On the basis of commercial elements, the cost of the solar engine in industrial production was evaluated at €525. With a lifespan of 20 years, with an annual average of $1760 kW h/m^2$ year, and with 0.15 efficiency, including the efficiency value of 0.5 for energy storage and destocking in an accumulator battery, we obtain a cost price of €0.099/ $kW h$, perfectly comparable with that of the $kW h$ produced by power stations.

6. Conclusion

Replacing the piston-cylinder system with metallic bellows solves the problem of air-tightness without energy dissipation. This innovation opens the possibility of transforming heat into work, with good real efficiency, using alternative heat engines of low power.

The solar heat engine studied, of low unitary mechanical power (100–3000 W), integrates this innovation. To guarantee its economic competitiveness, it includes a flat plate solar collector and a free-displacer Stirling engine. Its competitiveness is

not, however, guaranteed unless its dimensioning is optimised. To this end, notions of finite time thermodynamics are applied to a mathematical model, which takes into account the most significant penalising factors: differences in dynamic temperatures between the sources and the driving medium, rate of compression and regenerator imperfections.

This study demonstrates that the above innovation enables the 0.24 efficiency value of the best solar cells to be exceeded. Strictly from the point of view of efficiency, such systems are serious competitors to solar cells. In addition, the technological production of these machines enables a moderate cost price. Approximate figuring, which gives a kW h cost in the order of €0.01, allows us to think that this system is equally competitive with regard to power stations.

These elements should enable the heat option of exploiting solar electromagnetic radiation to become a possibility.

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